CID No: ________________________

IMPERIAL COLLEGE LONDON

Design Engineering MEng

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE71004 – Design of Visual Systems

SAMPLE PAPER 1

THE DURATION OF THIS EXAMINATION IS 90 MINUTES.

This paper contains EIGHT questions.
Full marks of the paper is 100 out of 100.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate how the examiners intend to distribute the marks for this paper.

Students are allowed to bring to the examination one double sided A4 sheet of handwritten information of their own choosing.

1) The intensity values r of an image are continuous between 0 and 255, and have the probability distribution function (PDF) of $p_r(r)$, where:

$p_r(r) = 3r^2/\,255^3$ for $0 \le r \le 255,$

$p_r(r) = 0$ for all other values of r.

The image is histogram-equalized with the intensity transformation function T, such that the new image intensity values is given by $s = T(r)$.

Derive the function $T(r)$.

[12]

Solution to Q1

This question tests student's understanding of the principle of histogram equalization. The test is on the following three key ideas: 1) the probability distribution function (PDF) is the normalised version of the intensity histogram of an image; 2) the cumulative distribution function is the integral of the probability distribution function (PDF); 3) the cumulative distribution function (CDF) is the intensity transformation required to achieve histogram equalization.

By definition, $s = T(r) = 0$ for all values r. outside the range of $[0, 255]$.

Therefore, the histogram equalization transformation $T(r)$ for the intensity range [0, 255] is the integral of the PDF function over the range of $[0, r]$, and scaled up by 255

Hence:

$$s = T(r) = 255\int_0^r p_r(x)\,dx = 255\int_0^r 3x^2/255^3 dx$$

$$= \frac{3}{255^2}\int_0^r x^2\,dx = \frac{3}{255^2} \times \frac{r^3}{3} = \frac{r^3}{255^2}.$$

So, s will also be in the range [0, 255].

[12]

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2) The image f and the filter kernel w are given in *Figure Q2*.

Compute the output image g, which is the filtered version of f with the kernel w using convolution. That is:

$$g = w * f.$$

You may assume that the input image is zero-padded with one extra row and column around the border, and that the output image is trimmed back to the original size.

[12]

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure Q2

Solution to Q2

This question tests student's understanding of filtering of an image with convolution.

Since the kernel is symmetrical, we don't need to flip it before performing convolution.

The answer is:

$$g = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 9 & 10 & 2 & 1 \\ 1 & 2 & 3 & 9 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

[12]

3) A colour image is displayed on a computer screen as three component colours: red (R), green (G) and blue (B), each represented by an 8-bit number. The background of the image has the following component values: R = 255, G = 255, B = 128. To an observer, what colour does the background of the image appears as?

[12]

Solution to Q3

This question tests student's understanding how R, G and B colours are mixed to form grayscale and determine the intensity.

$$255R + 255G\ + 128B = 128(R + G + B) + 128(R + G).$$

$128(R + G + B)$ is half grey. R+G combine to form yellow. Therefore, $128(R + G)$, is mid-intensity yellow. Overall, the background appears as pal yellow.

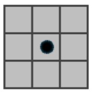
[12]

rgb(255,255,128)

4) Figure Q4 shows an image with dark pixel being the foreground and white pixel being the background.

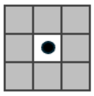
Sketch the result of eroding the image with each of the following structuring elements:

(a)



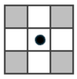
[5]

(b)



[5]

(c)



[5]

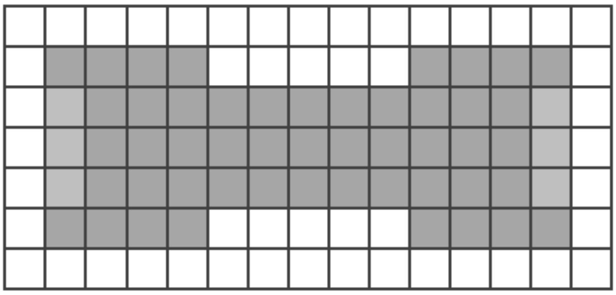
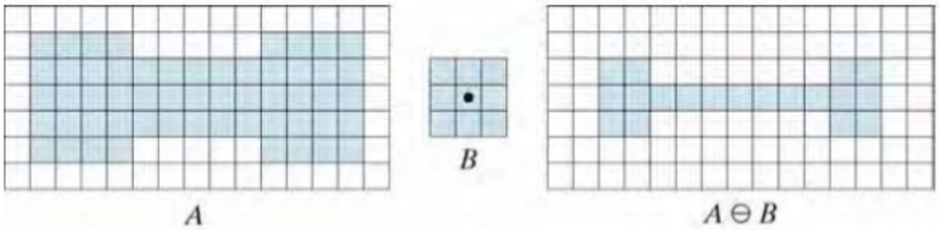


Figure Q4

Solution to Q4

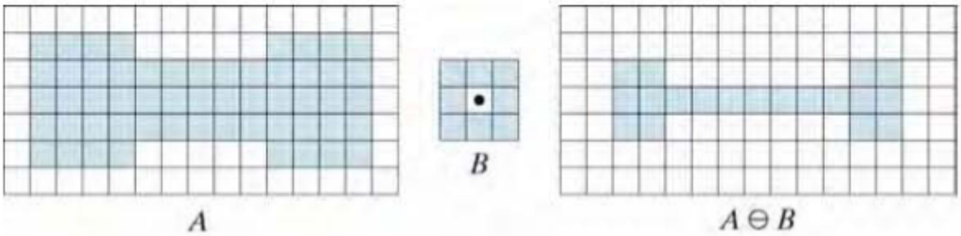
This question tests student's understanding of the erosion morphological operations.

a)



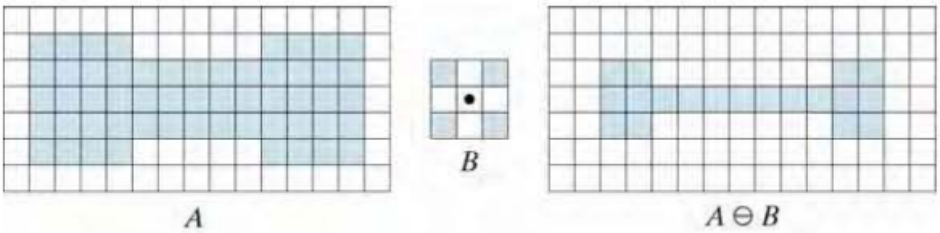
[5]

b)



[5]

c)



[5]

Despite all three solutions to be identical, one still need to work out each of the solutions.

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5) *Figure Q5* shows a colour photo of a shallow pond full of tadpoles. Note that due to the shallowness of the pond and the lighting condition, some tadpoles also appear as shadows.

Propose with justifications a sequence of steps you may use to process this image in order to automatically count the number of tapoles in the photograph.

[12]

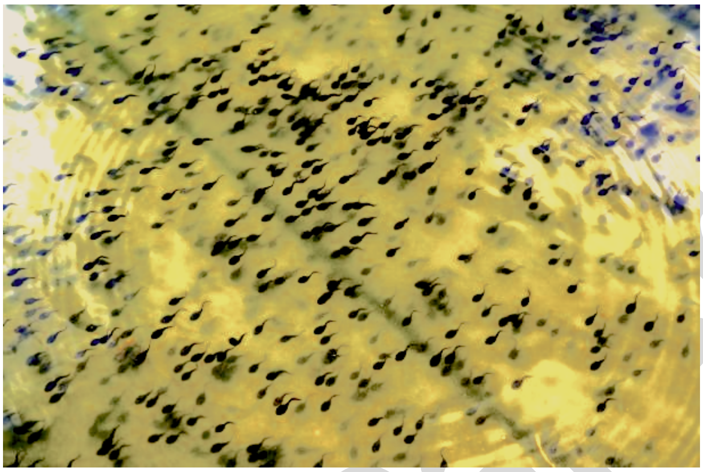


Figure Q5

Solution to Q5

This question tests student's ability to apply techniques they have learned to perform effective segmentation and object identification. There is right or wrong answer to this question because all in actual applications, one would try different techniques and find the right one for tadpole counting application.

Here is a possible answer:

- Since we do not need colour to count tadpoles, it would be sensible to convert this to a greyscale image.
- There is no significant noise in the image, therefore noise reductions is NOT required.
- The actual tadpole is much darker than the shadow tadpoles and the background, therefore some form of thresholding technique could be employed. May try to Otsu's method first to see if it works. If not try variable thresholding in different regions.
- The scale of the tadpoles are similar (i.e. they are similar in shape) and the main figure is substantially circular. We can create a template of tadpole using an average size blob of circular shape. With the template, we can produce the normalised cross correlation function of the image. Then use thresholding technical to identify the location of each tadpoles before counting them.

[12]

6) In Scale Invariant Feature Transform, scale invariant is achieved by calculating the "*sigma normalised 2nd derivative*" of blobs. *Figure Q6* shows three sizes of blobs in a 1–D image and the Gaussian kernel.

Sketch the results of the following:

(a) 1st derivative of the Gaussian kernel (same for all three).

[4]

(b) Applying th2nd derivative of the Gaussian to the three blobs.

[4]

(c) Applying the normalised 2nd derivative of the Gaussian to the three blobs.
.

[4]

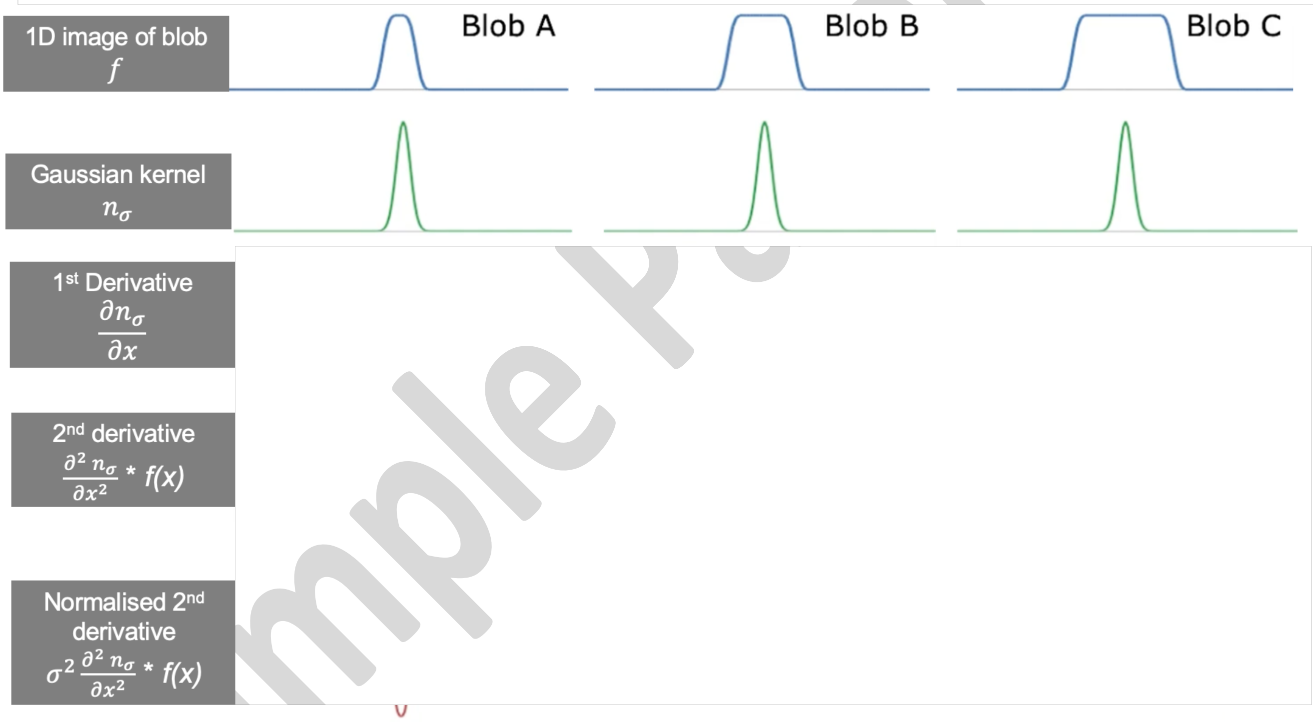
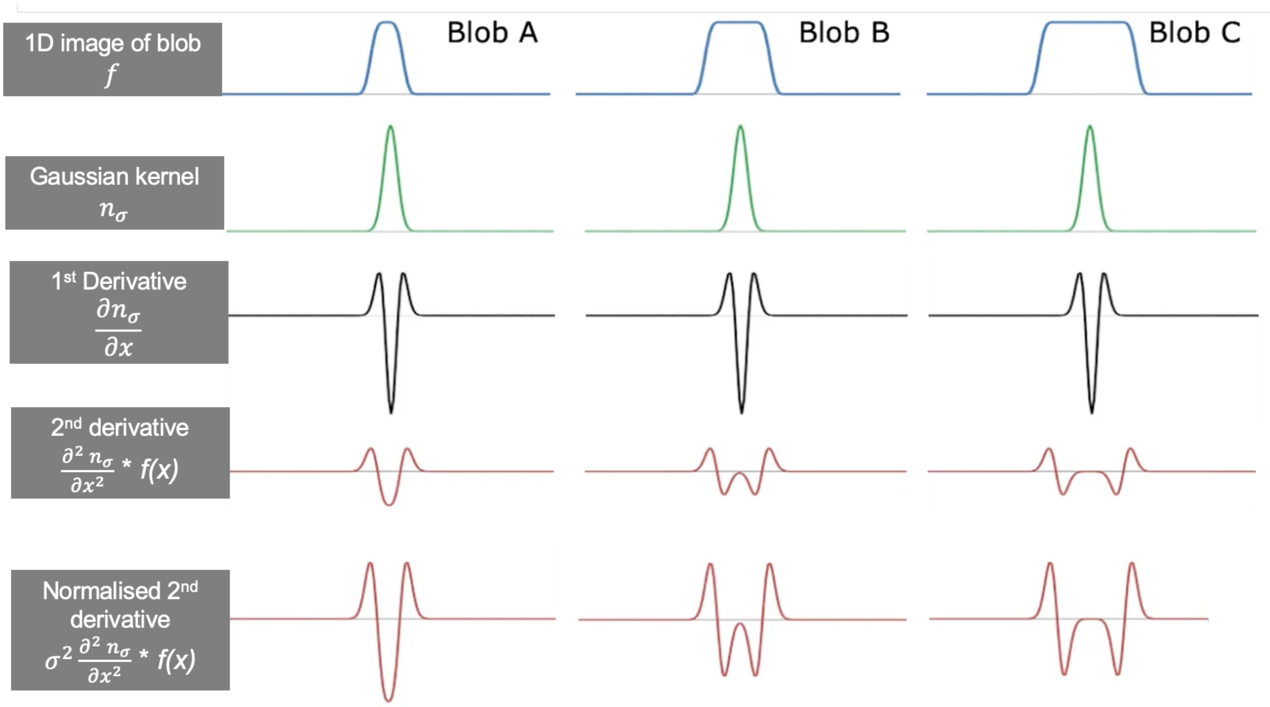


Figure Q6

Solution to Q6

This question tests students understanding of the initial part of the SIFT algorithm for feature matching. The answer is directly from Lecture 10 slide 19.



[12]

7) What is an image pyramid and why is it useful?

[5]

Briefly explain how an image pyramid can be constructed. State any assumptions used.

[5]

Solution to Q7

(a) An image pyramid is a representation of an image in different resolution. Each level of the pyramid represents a reduced resolution, normal half the resolution, of the previous level starting from the original image.

Image pyramids allow image processing operation such as image recognition, object detection and image segmentation to be performed at a resolution that provides the necessary accuracy with least computation.

[5]

(b) To construct an image pyramid, the original image is first lowpass filtered to avoid the introduction of unwanted artifacts (due to aliasing effect) before subsampling to generate the next level of image at a reduced resolution. The normal ratio of reduction is 2:1 in the linear dimension. I.e. a N x N image is reduced to N/2 x N/2. This is repeated for multiple levels until the lowest level of resolution image is produced at the top of the pyramid.

[5]

8. A contact lens has a concave surface on the side that will be in contact with the eye's cornea, however it can still behave as a convex lens due to the radius of curvature of the other side.

a) What is the focal length of such a lens, given an inner radius of curvature -8mm, and outer radius of curvature 7.5mm, thickness 0.5mm, and index of refraction $n = 1.33$? Restate the inverse focal length – the lens power – in diopters (inverse metres).

b) For a lens with the same thickness and focal length, what would the radius of curvature be if the lens was symmetric and convex? State to 3 s.f.

[15]

Lens Formula: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Lens Maker Formula: $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)d}{nR_1R_2}\right]$

Solution to Q7

a) Use lens maker formula, $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)d}{nR_1R_2}\right]$, giving,

$$\frac{1}{f} = (1.33-1)\left[\frac{1}{-8} + \frac{1}{7.5} - \frac{(1.33-1)0.5}{1.33 \times -8 \times 7.5}\right] \times 1000\ \mathrm{m}^{-1}$$
$$= 3.43\ \mathrm{m}^{-1}$$

And the focal length,

$$f = 291\text{mm}$$

b) Lens maker again, $\frac{1}{f} = (n-1)\left[\frac{2}{R} - \frac{(n-1)d}{nR^2}\right]$, rearrange as polynomial in R^{-1},

$$-\frac{(n-1)^2 d}{n} R^{-2} + 2(n-1)R^{-1} - f^{-1} = 0$$

Substitute values,

$$-0.0409R^{-2} + 0.660R^{-1} - 0.00343 = 0$$

Solve using quadratic formula,

$$R = 192\text{mm}$$

[15]

[END OF PAPER]